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THEORY AND METHODS OF SIGNAL PROCESSING

One-Stage Estimation of the Position of a Radio Source by a Wide-Base Passive System for an Unknown Signal Level

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Abstract—An optimal algorithm of the one-stage estimation of a wide-base passive system's emitter position in the case when the received signal level is unknown is presented. A Cramer–Rao matrix lower bound that describes the potential accuracy of estimation of both the informative parameters of the emitter coordinates and the noninformative ones (amplitudes of received signals) is derived. A simplified method for implementation of the optimal algorithm that allows measurements for a practice time acceptable for available computing facilities is proposed. The accuracy of the measurements made in accordance with this simplified algorithm is studied. It is shown that the proposed solutions allow implementation of the measurement process with an accuracy described by the Cramer–Rao lower bound.

INTRODUCTION

A wide-base passive system for the one-stage estimation of the emitter position is described in [1]. This system consists of receiving points (RP) separated in space and a central processing point (CPP) (Fig. 1). A useful signal model is selected as follows:

$$s_n(t) = a_n s(t - D_n),$$

where a_n is the amplitude factor specified by the attenuation factor of signal $s_n(t)$ when it is transmitted from the emitter (E) to the nth RP and D_n is the propagation time of signal $s(t)$ from E to the nth RP.

Here, the unknown parameters are a_n and D_n. In [1], a one-stage algorithm was synthesized under the assumption that the value of parameter a_n is known. From the viewpoint of practical implementation, this simplification of the problem can be treated as if there is some external procedure for estimating the value of a_n. In this case, the following questions arise: How will the complete algorithm of estimating coordinates look in conditions that amplitudes of the received signal are unknown? How will the Cramer–Rao lower bound be modified when new unknown parameters are added? How will the system implementing the proposed algorithm look in view of the capabilities of existing computing facilities?

In addition, the initial simplification of the problem by the artificial exclusion of one or other parameter leaves a feeling of incompleteness. In this respect, the solutions obtained in [2] appear more complete (analogues of the systems considered in [1, 2] are range-difference and direction-finding systems). The purpose of this work is to approach the existing solution obtained in [1] in the real electromagnetic environment.

1. THE ONE-STAGE ESTIMATION ALGORITHM AND THE CRAMER–RAO LOWER BOUND

Paper [1] presents a one-stage algorithm for estimating coordinates of an emitter. If an estimation problem with unknown amplitudes is included into the general problem, then, upon deriving expressions (9)–(12) in [1] and passing from the discrete to continuous time, we can write the logarithm of the normalizing factor as

$$\ln(C_{\text{norm}}) = (-T_{\text{obs}}\Delta_F N)\ln(2\pi) - J_3(\mathbf{a}).$$

Taking into account the symmetry of the spectra appearing under the integral sign with respect to the zero frequency, $\int_{-\infty}^{+\infty} X(f)df = 2\text{Re}\int_0^{+\infty} X(f)df$, the fol-

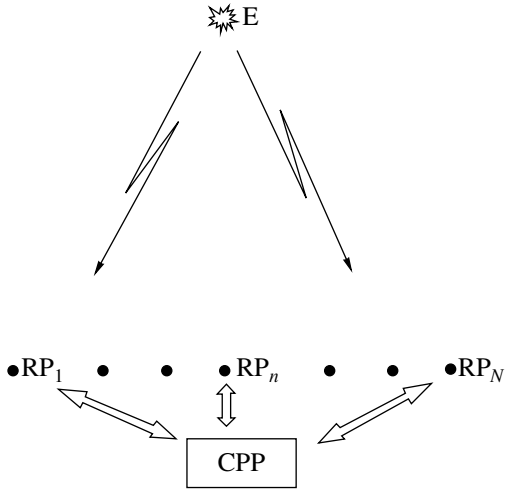

Fig. 1.

lowing transformation of the conditional probability density will take place:

$$\ln(P(\mathbf{U}|\mathbf{r},\mathbf{h},\mathbf{a})) = (-T_{\text{obs}}\Delta_F N)\ln(2\pi) - \sum_{n=1}^{N} T_{\text{obs}}\int_0^{\infty}\ln\left(\frac{1}{T_{\text{obs}}}G_{\xi n}(f)\right)df - J_{0n} + L(\mathbf{r},\mathbf{h},\mathbf{a}), \quad (1)$$

where

$$L(\mathbf{r},\mathbf{h},\mathbf{a}) = \left[\sum_{n=1}^{N} J_{1n}(a_n)\right] - J_3(\mathbf{a}) + 2\text{Re}(J_2(\mathbf{r},\mathbf{h},\mathbf{a})),$$

$$J_{0n} = \int_0^{\infty} G_{\xi n}^{-1}(f)U_n(f)U_n^*(f)df,$$

$$J_{1n}(a_n) = \int_0^{+\infty} W_n^2(f)U_n(f)U_n^*(f)df,$$

$$2\text{Re}(J_2(\mathbf{r},\mathbf{h},\mathbf{a})) = J_2(\mathbf{r},\mathbf{h},\mathbf{a}) + J_2^*(\mathbf{r},\mathbf{h},\mathbf{a}),$$

$$J_2(\mathbf{r},\mathbf{h},\mathbf{a}) = \sum_{k=1}^{N-1}\sum_{n=k+1}^{N}\int_0^{+\infty} W_n(f)W_k(f)U_n(f)U_k^*(f) \times \exp(j2\pi f(\tau_\eta^\Delta(\mathbf{r}) - \tau_\kappa^\Delta(\mathbf{r}) + \chi_\eta^\Delta - \chi_\kappa^\Delta))df$$

$$(\eta = n-1,\ \kappa = k-1,\ \tau_0^\Delta(\mathbf{r}) = \chi_0^\Delta \equiv 0),$$

$$J_3(\mathbf{a}) = T_{\text{obs}}\int_0^{\infty}\ln\left(1 + \sum_{k=1}^{N} a_k^2\frac{G_s(f)}{G_{\xi k}(f)}\right)df,$$

$\mathbf{U} = \|U_1(f), \ldots, U_N(f)\|^T$ is the vector of input signals; $\mathbf{r} = \|X_\text{r}, Y_\text{r}, Z_\text{r}\|^T$ are the coordinates of the emitter; $\mathbf{h} = \|\chi_1^\Delta, \ldots, \chi_{N-1}^\Delta\|^T$ is the vector of relative synchronization errors; $\mathbf{a} = \|a_1, \ldots, a_N\|^T$ is the vector of amplitudes of the input signal; T_{obs} is the observation time; $2\Delta_F$ is the frequency interval in which the signal is located; N is the number of the RPs; $G_s(f)$ and $G_{\xi n}(f)$ are the power spectral densities of the signal and noise, respectively; $W_n(f) = \dfrac{a_n}{G_{\xi n}(f)}\sqrt{\dfrac{G_s(f)}{1 + \sum_{k=1}^{N} q_k(f)}}$; and $q_n(f) = \dfrac{a_n^2 G_s(f)}{G_{\xi n}(f)}$ is the signal-to-noise ratio in the nth channel.

The measurement algorithm given by expression (15) in [1] can be written as

$$L(\hat{\mathbf{r}},\hat{\mathbf{h}},\hat{\mathbf{a}}) = \max_{\mathbf{r}\in\mathbf{R},\,\mathbf{h}\in\mathbf{H},\,\mathbf{a}\in\mathbf{A}} L(\mathbf{r},\mathbf{h},\mathbf{a}). \quad (2)$$

The possible implementation of the position-finding system operating in accordance with algorithm (2) is considered below.

Since the number of estimated parameters has increased, the number of dimensions of Cramer–Rao matrix lower bound $\mathbf{\Phi}_{\text{CR}}$ (and, respectively, of Fisher's information matrix $\mathbf{\Phi}$) have also increased. In the described case, these matrices can be written as

$$\mathbf{\Phi}_{\text{CR}} = \begin{Vmatrix} \mathbf{\Phi}_{\text{CR}}^{(13)} & \mathbf{0}_{\Omega+N-1}^{N} \\ \mathbf{0}_{N}^{\Omega+N-1} & \mathbf{\Phi}_{\text{CR}}^{(4)} \end{Vmatrix} = \mathbf{\Phi}^{-1},$$
$$\mathbf{\Phi} = \begin{Vmatrix} \mathbf{\Phi}_{13} & \mathbf{0}_{\Omega+N-1}^{N} \\ \mathbf{0}_{N}^{\Omega+N-1} & \mathbf{\Phi}_{4} \end{Vmatrix}, \quad (3)$$

where $\mathbf{\Phi}_{\text{CR}}^{(13)} = (\mathbf{\Phi}_{13})^{-1}$, $\mathbf{\Phi}_{\text{CR}}^{(4)} = (\mathbf{\Phi}_4)^{-1}$, $\mathbf{\Phi}_{13} = \begin{Vmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 \\ \mathbf{\Phi}_2^T & \mathbf{\Phi}_3 \end{Vmatrix}$ is the Fisher's matrix described by expression (21) in [1], and $\mathbf{0}_x^y$ is the zero matrix with x rows and y columns.

Matrix $\mathbf{\Phi}_4$ consists of elements $\mathcal{F}_{kn}^{(4)} = -\text{M}\left(\dfrac{\partial^2 L(\mathbf{r},\mathbf{h},\mathbf{a})}{\partial a_k \partial a_n}\right)_{(\mathbf{v})}$ ($k,\ n = \overline{1,N}$; $\text{M}(X)_{(\mathbf{v})} \equiv \text{M}(X)_{\left(\begin{smallmatrix}\mathbf{r}=\hat{\mathbf{r}}\\ \mathbf{h}=\hat{\mathbf{h}}\\ \mathbf{a}=\hat{\mathbf{a}}\end{smallmatrix}\right)}$), where

$$\text{M}\left(\frac{\partial^2 L(\mathbf{r},\mathbf{h},\mathbf{a})}{\partial a_k \partial a_n}\right)_{(\mathbf{v})} = \int_0^{+\infty} G_s(f)\{\partial^2 Q(f)/\partial a_k\partial a_n \cdot \text{M}[S(f)]_{(\mathbf{v})} + \text{M}[\partial S(f)/\partial a_k]_{(\mathbf{v})}\cdot\partial Q(f)/\partial a_n + \text{M}[\partial S(f)/\partial a_n]_{(\mathbf{v})}\cdot\partial Q(f)/\partial a_k + \text{M}[\partial^2 S(f)/\partial a_n\partial a_k]_{(\mathbf{v})}\cdot Q(f)\}df - \partial^2 J_3(\mathbf{a})/\partial a_n\partial a_k,$$

$$Q(f) = \left(1 + \sum_{k=1}^{N} q_k(f)\right)^{-1},$$

$$S(f) = \sum_{k=1}^{N}\sum_{n=1}^{N} a_n a_k G_{\xi n}^{-1}(f) G_{\xi n}^{-1}(f) U_n(f) U_k^*(f)$$

$$\times \exp(j2\pi f(\tau_\eta^\Delta(\mathbf{r}) - \tau_\kappa^\Delta(\mathbf{r}) + \chi_\eta^\Delta - \chi_\kappa^\Delta))$$

$$(\eta = n-1,\ \kappa = k-1,\ \text{and}\ \ \tau_0^\Delta(\mathbf{r}) = \chi_0^\Delta \equiv 0),$$

$$\partial Q(f)/\partial a_k = -2a_k G_s(f) G_{\xi k}^{-1}(f)\left(1 + \sum_{b=1}^{N} q_b(f)\right)^{-2},$$

$$\partial^2 Q(f)/\partial a_k \partial a_n = 8a_k a_n G_s^2(f) G_{\xi k}^{-1}(f) G_{\xi n}^{-1}(f) \quad (4)$$

$$\times\left(1 + \sum_{b=1}^{N} q_b(f)\right)^{-3} - 2\delta_{kn} G_s(f) G_{\xi k}^{-1}(f)\left(1 + \sum_{b=1}^{N} q_b(f)\right)^{-2},$$

$$\mathrm{M}[S(f)]_{(\mathbf{v})} = T_{\mathrm{obs}} G_s(f) \sum_{k=1}^{N}\sum_{n=1}^{N} a_n^2 a_k^2 G_{\xi n}^{-1}(f) G_{\xi k}^{-1}(f)$$

$$+ \sum_{b=1}^{N} a_b^2 G_{\xi b}^{-1}(f),$$

$$\mathrm{M}[\partial S(f)/\partial a_k]_{(\mathbf{v})} = 2T_{\mathrm{obs}} \sum_{b=1}^{N} a_b G_{\xi b}^{-1}(f) G_{\xi k}^{-1}(f)$$

$$\times (a_b a_k G_s(f) + \delta_{bk} G_{\xi k}(f)),$$

$$\mathrm{M}[\partial S(f)/\partial a_k \partial a_n]_{(\mathbf{v})}$$

$$= 2T_{\mathrm{obs}} G_{\xi k}^{-1}(f) G_{\xi n}^{-1}(f)(a_k a_n G_s(f) + \delta_{kn} G_{\xi n}(f)),$$

and

$$\partial^2 J_3(\mathbf{a})/\partial a_k \partial a_n$$

$$= T_{\mathrm{obs}} \int_0^\infty \left[-2a_k a_n G_s^2(f) G_{\xi k}^{-1}(f) G_{\xi n}^{-1}(f)\left(1 + \sum_{b=1}^{N} q_b(f)\right)^{-2}\right.$$

$$\left. + \delta_{kn} G_s(f) G_{\xi k}^{-1}\left(1 + \sum_{b=1}^{N} q_b(f)\right)^{-1}\right] df.$$

To implement procedure (2), the coordinate computer should have the information on parameters of the signal and the noise. With the advent of one or another heuristic algorithm that permits measurements of some parameters, the following questions arise: how well does the proposed algorithm operate and to what degree do measurement errors of this parameter influence the coordinate estimation accuracy? The expressions derived above provide answers to these questions in a general form. At the same time, it is expedient to consider a particular example of implementation of the coordinate measurement algorithm that allows one to see the influence of one measurement (the signal amplitudes) on another (the emitter coordinates).

2. IMPLEMENTATION OF THE OPTIMAL ALGORITHM

In [1], an algorithm for estimating emitter coordinates was synthesized and analyzed on the assumption that the power spectral densities of the signal and the noise are known. The measurement of these parameters was not included in the circle of problems solved in [1]. This study concretely renders the problem of estimating additional parameters of the received signals used in the implementation of the coordinate measurement algorithm.

The basic problem that must be solved is the problem of implementing a rather complicated procedure (2). The estimation algorithm is complicated mainly owing to an increase in the number of dimensions of the space in which we seek the maximum of function $L(\mathbf{r}, \mathbf{h}, \mathbf{a})$. Below we propose a method allowing estimation of the emitter coordinates in accordance with (2).

To find the maximum of the likelihood function (LF), the following method is the most efficient. Initially, the primary estimation of **a** is performed. For this purpose, some simplifications are made. At the beginning, it is assumed that the nth RP operates independently. As a result, the likelihood function ceases to depend on **r** and **h**. In this case, we can write

$$L(a_n) = J_{1n}(a_n) - J_3(a_n)$$

$$= \int_0^{+\infty} a_n^2 G_s(f) G_\xi^{-2}(f)(1 + a_n^2 G_s(f) G_\xi^{-1}(f))^{-1}$$

$$\times U_n(f) U_n^*(f) df - T_{\mathrm{obs}} \int_0^\infty \ln(1 + a_n^2 G_s(f) G_\xi^{-1}(f)) df.$$

Having taken a derivative of $L(a_n)$, we obtain

$$\partial L(a_n)/\partial a_n = 2a_n \int_0^{+\infty} G_s(f) G_\xi^{-1}(f)$$

$$\times (1 + a_n^2 G_s(f) G_\xi^{-1}(f))^{-1}$$

$$\times \{(G_\xi(f) + a_n^2 G_s(f))^{-1} U_n(f) U_n^*(f) - T_{\mathrm{obs}}\} df.$$

In the next step, which allows simplification of the estimation procedure for the initial approximation to a_n, the power spectral densities depending on fre-

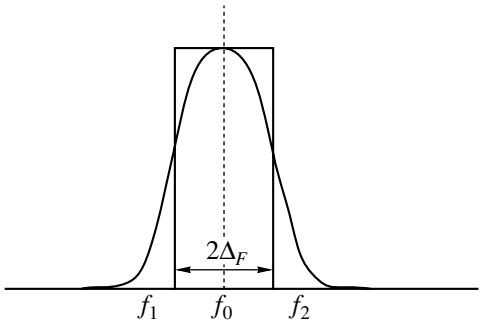

Fig. 2.

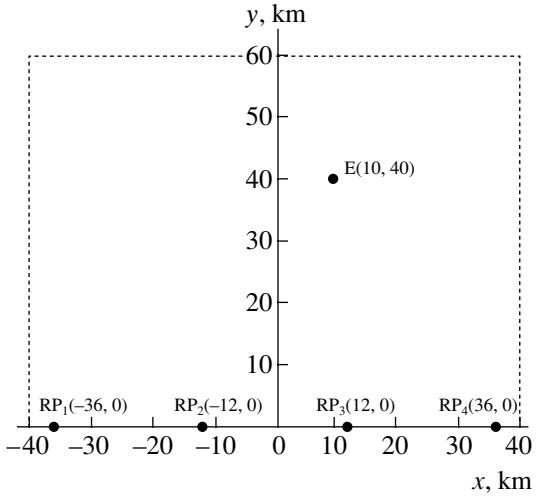

Fig. 3.

quency f are replaced by rectangular functions with width $2\Delta_F$ (see Fig. 2). In this case, $G_s(f) \longrightarrow G_s$ and $G_\xi(f) \longrightarrow G_\xi$. If the derivative of the LF is equated to zero and solutions for which $a_n = 0$ and $a_n = \infty$ are excluded, we obtain

$$\hat{a}_n = \left\{\left[\int_0^{+\infty} U_n(f)U_n^*(f)df/(2\Delta_F T_{\text{obs}}) - G_\xi\right]/G_s\right\}^{1/2}. \quad (5)$$

Substituting the values obtained from (4) into (2), we scan spaces **R** and **H** with the goal of estimating vectors **r** and **h** by finding the maximum of the LF given in (1).

Now, let us determine how acceptable the first approximation obtained from (5) is.

In accordance with this rule, the amplitude-measurement error can be described by using (4) and assuming that $N = 1$. In this case, the Cramer–Rao matrix lower bound degenerates into a scalar quantity equal to σ_a^2. Here,

$$\sigma_a = \frac{1+q}{2q\sqrt{2\Delta_f T_{\text{obs}}}}, \quad (6)$$

where $q = aG_s/G_\xi$ is the signal-to-noise ratio.

Let us consider how much rule (6) loses in the accuracy of the amplitude measurement with respect to the optimal procedure (2). To clarify the analysis, we consider the example with $N = 2$. We then obtain for (3)

$$\mathbf{\Phi}_{\text{CR}}^{(4)} = \left\| \begin{array}{cc} \sigma_{a_1}^2 & \sigma_{a_1 a_2}^2 \\ \sigma_{a_1 a_2}^2 & \sigma_{a_2}^2 \end{array} \right\|, \quad (7)$$

where

$$\sigma_{a_1}^2 = \mathcal{F}_{22}^{(4)}/\det, \quad \sigma_{a_2}^2 = \mathcal{F}_{11}^{(4)}/\det,$$

$$\det = \mathcal{F}_{11}^{(4)}\mathcal{F}_{22}^{(4)} - \mathcal{F}_{12}^{(4)}\mathcal{F}_{21}^{(4)},$$

$$\mathcal{F}_{11}^{(4)} = \rho(2a_1^2 + a_2^2 + g a_1^2 a_2^2 + g a_2^4),$$

$$\mathcal{F}_{22}^{(4)} = \rho(2a_2^2 + a_1^2 + g a_1^2 a_2^2 + g a_1^4),$$

$$\mathcal{F}_{12}^{(4)} = \mathcal{F}_{21}^{(4)} = \rho a_1 a_2 (1 - g(a_1^2 + a_2^2)),$$

$$\rho = -4\Delta_f T_{\text{obs}} g^2 (1 + g(a_1^2 + a_2^2))^{-2}, \text{ and } g = G_s/G_\xi.$$

As an example, the table summarizes the dependence of relative errors $\sigma_{a_1}/\sigma_{a_1}^{\text{q}}$ and $\sigma_{a_2}/\sigma_{a_2}^{\text{q}}$ on parameters a_1 and a_2, where $\sigma_{a_1}^{\text{q}}$, described by expression (6), is the mean square deviation (MSD) of the amplitude-measurement error in quasi-optimal algorithm (5). In

Dependence of relative errors $\sigma_{a_1}/\sigma_{a_1}^{\text{q}}$ and $\sigma_{a_2}/\sigma_{a_2}^{\text{q}}$ on parameters a_1 and a_2

a_1, dB	a_2, dB			
	–5	0	5	10
–5	0.9250 / 0.9250	0.8166 / 0.9928	0.7232 / 0.9998	0.6765 / 1.0000
0	0.8166 / 0.9928	0.9682 / 0.9682	0.9250 / 0.9983	0.8907 / 1.0000
5	0.7232 / 0.9998	0.9250 / 0.9983	0.9928 / 0.9928	0.9832 / 0.9998
10	0.6765 / 1.0000	0.8907 / 1.0000	0.9832 / 0.9998	0.9990 / 0.9990

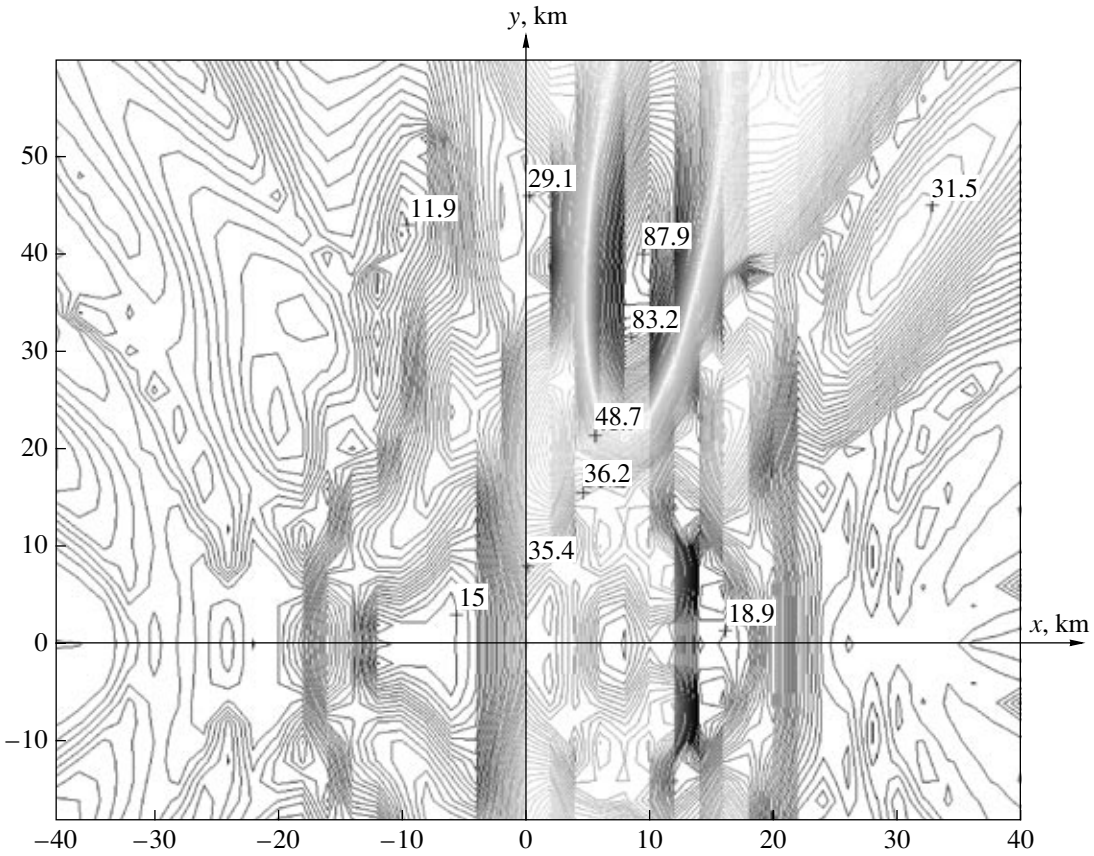

Fig. 4.

this case, ratio $\sigma_{a_1}/\sigma^{q}_{a_1}$ is indicated in the upper part of each cell of the table and ratio $\sigma_{a_2}/\sigma^{q}_{a_2}$ is presented in the lower part of the cell. As can be seen, when the signal level increases, the optimal and quasi-optimal algorithms tend to be equivalent from the viewpoint of the signal-amplitude measurement accuracy.

Let us now directly consider the coordinate-measurement procedure by using the above measurement-simplification method. Figure 3 shows the position-finding system containing four RPs.

The emitter coordinates are measured as follows. Initially, vector $\mathbf{a}_h$ is measured using heuristic procedure (5). Then, a coarse scan along coordinates x and y of the space in which the emitter can be located is performed with some step (in our case, 2 km). In this case, at each scan point $\mathbf{r}_{sc}$, the LF maximum is calculated using a numerical method (in our case, this is the simplex method [3]):

$$L(\hat{\mathbf{h}}_{sc}) = \max_{\mathbf{h} \in \mathbf{H}} L(\mathbf{h}, \mathbf{r}_{sc}, \mathbf{a}_h), \qquad (8)$$

where vector $\hat{\mathbf{h}}$ is the noninformative parameter. To explain the operation of the intermediate procedure (8), we can cite, as an example, the case of $N = 2$. Here, the total LF is a slow process filled with a high-frequency oscillation (its plot resembles that of an amplitude-modulated signal). The purpose of the complete algorithm is to jump over the tops of the harmonics. During the scan, it is difficult to jump to the tops. Hence, at each new step, intermediate procedure (8) needs to be available to push out the sought-for value of the LF to any nearest top. The measured coordinate values are obtained as if they are discrete, but, since it was assumed in the problem formulated in [1] that $2\Delta_F \ll f_0$, then it is almost impossible to notice this discreteness.

In accordance with (8), we can now plot dependence $L_\Phi(\mathbf{r})$. Here subscript Φ signifies the phase self-tuning in the case of selecting a new point in space $\mathbf{H}$ of the likelihood function; i.e., $L_\Phi(\mathbf{r}) = L(\mathbf{r}, \mathbf{h}, \mathbf{a})$ when $\mathbf{h} = \hat{\mathbf{h}}$ and $\mathbf{a} = \mathbf{a}_h$.

Figure 4 shows an example of the LF plotted for discrete values of vector $\mathbf{r} - \mathbf{r}_{sc}$. The input data are as follows. The observation time is 1.3 ms, the power spectral densities of the signal and noise are uniform in a 40-kHz frequency band, signal-to-noise ratio $G_s/G_\xi = 1$, and signal amplitude vector $\mathbf{a} = (1, 1, 1, 1)$. For this case, the LF maximum is 87.9. As can be seen from the figure, the global maximum is near true coordinate values (10, 40).

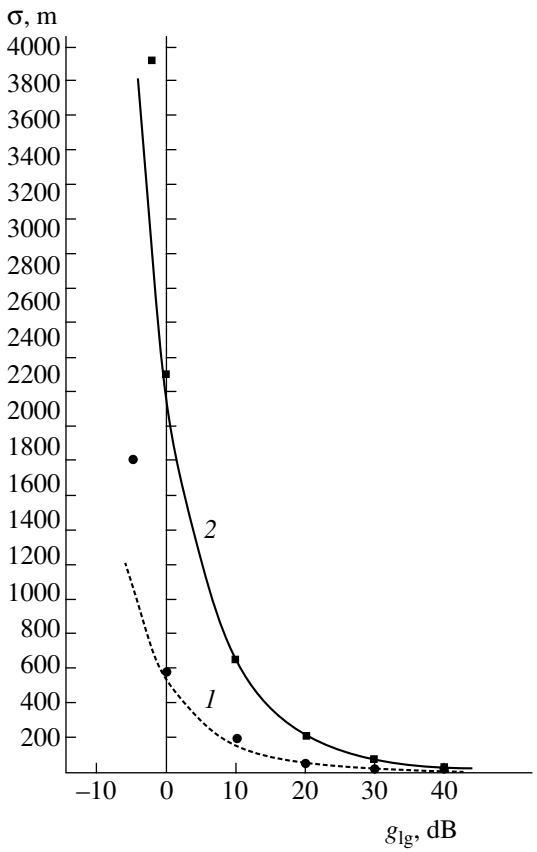

Fig. 5.

After the coarse LF value and, respectively, approximate estimation of the emitter location are found, the coordinates are additionally determined by searching for the maximum of function $L_\Phi(\mathbf{r})$ using a numerical (simplex) method.

The proposed algorithm of searching for an estimation of the emitter coordinates is based on some heuristic procedures that allow a substantial reduction of the measurement time. In order to understand, at least in the first approximation, how successful the proposed solutions are, it is expedient to analyze the obtained results by performing a simulation on a personal computer.

Figure 5 shows a graph of MSDs versus the signal-to-noise ratio. Curves *1* and *2* are, respectively, dependencies of σ_x and σ_y on g_{lg}, where $g_{lg} = 20\log(g_{lin})$ is the value characterizing the general signal level ($\mathbf{a} = g_{lin} \cdot (1, 1, 1, 1)$), and σ_x and σ_y are the theoretical MSDs along axes x and y, respectively, calculated in accordance with expression (22) in [1] which, for the 2D space, can be written as

$$\mathbf{\Phi}_{CR}^{(1)} = \left\| \begin{array}{cc} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{array} \right\|.$$

In the figure, circles and squares mark estimated MSD values $s_x = \left(\frac{1}{\mathcal{N}-1}\sum_{n=1}^{\mathcal{N}}(\hat{X}_n - X_r)^2\right)^{1/2}$ and $s_y = \left(\frac{1}{\mathcal{N}-1}\sum_{n=1}^{\mathcal{N}}(\hat{Y}_n - Y_r)^2\right)^{1/2}$, respectively, where $\mathcal{N}$ is the number of tests and $\hat{X}_n$ and $\hat{Y}_n$ are the estimated coordinates of the nth measurement.

As follows from the figure, for the chosen initial data, the proposed measurement algorithm is quite stable and its error is well described by the Cramer–Rao lower bound.

CONCLUSIONS

A one-stage algorithm of estimating coordinates by a wide-base passive system for an unknown signal level is proposed. In addition to [1], this study yields a solution that depends less on *a priori* knowledge of parameters of the received signal. As a parameter whose value is virtually impossible to foresee, the signal level at the receiving point was selected. In spite of an increase in the number of dimensions, a quite operable measurement algorithm was obtained in this work. In this case, its operating accuracy is close to the potential accuracy described by the Cramer–Rao lower bound.

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